Using Off-Resonance Laser Modulation for Beam-Energy-Spread Cooling in Generation of Short-Wavelength Radiation

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To improve temporal coherence in electron beam based light sources, various techniques employ frequency up conversion of external seed sources via electron beam density modulation; however, the energy spread of the beam may hinder the harmonic generation efficiency. In this Letter, a method is described for cooling the electron beam energy spread by off-resonance seed laser modulation, through the use of a transversely dispersed electron beam and a modulator undulator with an appropriate transverse field gradient. With this novel mechanism, it is shown that the frequency up-conversion efficiency can be significantly enhanced. We present theoretical analysis and numerical simulations for seeded soft x-ray free-electron laser and storage ring based coherent harmonic generation in the extreme ultraviolet spectral region.

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The availability of high brightness and short-wavelength radiation, especially x-ray pulses, is of great interest. X-ray pulses enable the simultaneous probing of both the ultrasmall and the ultrafast worlds and continue to revolutionize the understanding of matter. Therefore, synchrotron radiation (SR) light sources and free-electron lasers (FELs) based on advanced particle accelerators are being developed worldwide to satisfy the dramatically growing demands in the material and biological sciences [1]. The fundamental process of SR and FEL sources usually involves a relativistic electron beam passing through a transverse periodic magnetic field, e.g., the undulator, and generating electromagnetic radiation ranging from the infrared to hard x-ray regions, depending on the electron beam energy and the undulator period and strength. More recently, the successful user operation of the first FEL facilities [2–5] in soft and hard x-ray regimes announced the birth of the x-ray laser. Currently, the light source community is continuing to develop more sophisticated schemes in pursuit of e.g., full coherence [6–18], fast polarization switch [19–22], and compact x-ray configurations [23–26].

Coherence describes all properties of the correlation between physical quantities of a single wave, or between several waves or wave packets. It is widely used in any field that involves waves, such as acoustics, electrical engineering, neuroscience, and quantum mechanics. Spatial coherence describes the correlation between waves at different position, which is naturally ensured in SR and FEL sources. Temporal coherence describes the predictable relationship between waves observed at different times, and is usually obtained in SR sources by purifying the noisy spectra with crystal monochromator. As the leading lasing mode of the hard x-ray FEL, self-amplified spontaneous emission [27] starts from the initial shot noise of the electron beam and results in radiation with poor temporal coherence. Recently it was demonstrated in the Linac Coherent Light Source [3] that the temporal coherence of self-amplified spontaneous emission could be significantly improved by the configuration of self-seeding [6].

Alternatively, in order to generate fully coherent radiation, various seeded FEL schemes [9–18] have been proposed and intensively studied around the world. One of the initial seeded FEL configurations is high gain harmonic generation (HGHG) [9], which has been perfectly demonstrated in the visible and ultraviolet region [4,10,11]. Unfortunately, the standard HGHG suffers an essential drawback that a single stage allows only a limited frequency multiplication factor. Therefore, a multistage HGHG approach [12–14] was proposed for short-wavelength production from an ultraviolet seed wavelength; however, this leads to a rather complicated overall design. Meanwhile, novel concepts are under development, e.g., echo-enabled harmonic generation [15] which has been experimentally demonstrated at the third, fourth, and seventh harmonics of the seed laser [16–18], and in principle could work efficiently at several tens of harmonic number in a single stage.

Recently, the use of a transverse gradient undulator was proposed to greatly reduce the effects of large beam energy spread from laser-plasma accelerators and ultimate storage rings for driving short-wavelength FELs [24,28]. Inspired by these earlier works, we point out in this Letter a new physical mechanism of using a transverse gradient undulator in seeded configurations for strongly enhancing the frequency up-conversion efficiency. Theoretical analysis and numerical simulations are used to demonstrate that the local energy spread at a certain phase relative to the seed laser wave can be suppressed significantly, thus delivering unprecedented frequency up-conversion efficiency even at several tens of harmonic order. In order to clearly
illustrate the new mechanism, we first review the conventional method to modulate the beam current in the standard-HGHG technique.

The standard-HGHG setup is shown in Fig. 1(a). Generally, the pulse length of the electron beam is much longer than the seed wavelength \( \lambda_s \), and the beam current variation over the distance of several laser wavelengths can be neglected. Thus, we locally assume a longitudinally uniform electron beam and Gaussian beam energy distribution with the average energy \( \gamma_0 mc^2 \) and the energy spread \( \delta \). The external laser and the electron beam interact with each other in the modulator undulator to give a sinusoidal energy variation in the electron beam energy. Upon passing through the dispersive chicane, the electron beam’s energy modulation is converted into density modulation, which contains frequency components at harmonics of the seed laser, and can be characterized by the bunching factor:

\[
\frac{K(x)}{K_0} = 1 + \alpha x = 1 + \alpha \eta \frac{\gamma - \gamma_0}{\gamma_0}. \tag{2}
\]

For a given wavelength of the seed laser, the resonant beam energy should be written as

\[
\gamma_r(x) = \gamma_0 + \alpha \eta \frac{K^2_0}{K^2_0 + 2} (\gamma - \gamma_0). \tag{3}
\]

Now we consider the longitudinal dynamics of the resonant electron (\( \gamma'_0, \theta_0 \)) and an arbitrary electron (\( \gamma', \theta_0 \)) at the exit of the modulator undulator, which is the electron (\( \gamma_0, \theta_0 \)) and (\( \gamma, \theta_0, \Delta \varphi \)) at the entrance of the modulator undulator, respectively. \( \Delta \varphi \) is the phase advance of the off-resonance electron with respect to the resonant one. It can be given by the phase equation of the modulator undulator:

\[
\Delta \varphi = 4\pi N \left( \frac{\gamma - \gamma_r}{\gamma_0} \right), \tag{4}
\]

where \( N \) represents the period number of the modulator. Then for small \( \theta_0 \), one has

\[
\gamma'_0 = \gamma_0 - \Delta \gamma \sin \theta_0 = \gamma_0 - \Delta \gamma \theta_0,
\]

\[
\gamma' = \gamma - \Delta \gamma \sin(\theta_0 - \Delta \varphi / 2) = \gamma - \Delta \gamma (\theta_0 - \Delta \varphi / 2), \tag{5}
\]

where \( \Delta \varphi / 2 \) is from the phase average of the off-resonance electrons in the modulator undulator. Combining Eqs. (4) and (5), we can easily derive that

\[
\frac{\gamma' - \gamma'_0}{\gamma - \gamma_0} = 1 - \frac{2\pi N \Delta \gamma}{\gamma_0} \left( \frac{\alpha \eta K^2_0}{K^2_0 + 2} - 1 \right). \tag{6}
\]

Equation (6) illustrates a universal scaling of the off-resonant laser modulation as a tool of beam energy spread cooling. It is clear that, in the standard-HGHG setup, the beam energy spread is amplified by a factor of \( 2\pi N \Delta \gamma / \gamma_0 \), which is usually a relatively small number. When we increase the \( \alpha \eta \) product and make the right-hand side of Eq. (6) to be unity, the beam energy spread is maintained because every electron satisfies the FEL resonant condition, as in Ref. [24]. If one further increases the \( \alpha \eta \) product properly, the right-hand side of Eq. (6) can be adjusted to zero, then the electron beam energy spread will be fully cooled; thus, the frequency up-conversion efficiency will be dramatically enhanced. This is the working regime of the cooled HGHG. Then according to Eq. (1), the bunching factor in the cooled HGHG is mainly determined by the Bessel function term, which can be optimized by the longitudinal dispersion of the chicane in Fig. 1(b). It means that for a moderate beam energy modulation, the bunching factor of the \( n \)th harmonic is about \( 0.67/n^{1/3} \).

In order to clearly illustrate the essence of the cooled HGHG, GENESIS1.3 [31] simulations using minute transverse emittance and beam size, i.e., 1D simulations, were carried out on the basis of the parameters of the Shanghai soft x-ray FEL test facility (SXFEL) [32]. The baseline

FIG. 1 (color online). A standard-HGHG system (a) consists of a modulator undulator and a dispersive section. The proposed cooled-HGHG scheme (b) includes a dogleg, a modulator undulator with transverse gradient, and a chicane.
The design of SXFEL is a two-stage HGHG from 265 to 8.8 nm, while here we propose to reach 8.8 nm by one stage of cooled HGHG. The parameters used are slice energy spread of 84 keV, beam energy of 0.84 GeV, peak current of 600 A, modulator period length of 80 mm, and period number of 12. We introduce a maximum energy modulation of 500 keV by the 265 nm seed laser in the modulator, i.e., 6 times of the initial beam energy spread. Then a 2D scan of the 30th harmonic bunching factor, as a function of $\eta$ and $\alpha$, was carried out. The result in Fig. 2 demonstrates a stable optimal zone with maximum bunching factor over 20%. It is found that the optimal relation is approximately $\alpha \eta \approx 25$ from the simulation, while Eq. (6) indicates an optimal value of $\alpha \eta = 24$. Considering the rough assumptions in the theoretical analysis, it is reasonably consistent with the simulation results.

With the optimized parameters above, Fig. 3 shows the longitudinal phase space evolution of the beam as it travels through the standard HGHG and the proposed cooled HGHG. These pictures demonstrate a simple physical mechanism behind the off-resonance laser modulation effect. The electron beam energy spread cooling indicated in Eq. (6) corresponds to a merger phenomenon around the center of the longitudinal beam phase space, while a broadening effect occurs at the two flanks. Considering that most of the electrons are concentrated around the center, the density modulation will be significantly enhanced in the cooled HGHG.

The above results are corroborated by the data shown in Fig. 4, where the maximum achievable bunching at different harmonics is plotted. Figure 4 also shows the results from the standard HGHG for comparison. These results

![Figure 2](https://example.com/fig2.png)

**FIG. 2** (color online). Optimization of the transverse gradient $\alpha$ of the modulator and the transverse dispersion $\eta$ of the dogleg by 1D simulation, in order to find the optimal bunching factor of the 30th harmonic for the cooled HGHG.

![Figure 3](https://example.com/fig3.png)

**FIG. 3** (color online). The phase space after the modulator undulator (top left) and the dispersive chicane (top right) in the standard HGHG and the phase space after the modulator undulator (bottom left) and the dispersive chicane (bottom right) in the cooled HGHG. The blue, green, and red, respectively, represent the electrons with high, medium, and low energy at the beginning of the modulator undulator.
were calculated by optimizing the longitudinal dispersion for each harmonic while using the phase space shown in Fig. 3. While the bunching factor exponentially decreases as the harmonic number increases for standard HGHG, it can be well maintained in the cooled HGHG scheme, e.g., over 15% even at the 50th harmonic.

For a realistic electron beam, the intrinsic horizontal beam size $\sigma_x$ results in an effective energy spread of $\gamma \sigma_x / \eta$ in the modulator, which cannot be fully cooled by the proposed method. The horizontal beam size $\sigma_x$ is determined by the normalized emittance $\varepsilon_x$ and the $\beta$ function. For a short modulator, it is reasonable to take $\beta = N \lambda / 2$ and $\sigma_x^2 = \varepsilon_x N \lambda / 2 \gamma$. Thus a rigorous derivation of the cooled HGHG gives a straightforward bunching factor as

$$b_n = e^{-(n^2 D^2 / 2)(\gamma^2 \sigma_x^2 / \eta^2)} J_n(n \Delta \gamma).$$

(7)

Figure 5(a) shows the bunching factor dependences on the horizontal beam emittance $\varepsilon_x$, according to Eq. (7). It indicates that the remarkable frequency up-conversion efficiency of the cooled HGHG could be well preserved by using a large dispersion $\eta$ when the beam emittance increases. Taking into account the seed laser diffraction and the transverse beam dynamics, the theoretical results of Eq. (7) have been fully confirmed by 3D simulation using the above SXFEL parameters. Generally, a further minus $\eta$ section should be introduced for removing the transverse beam size dependence on the initial beam energy spread. However, it may increase the beam size because of the large external beam energy spread induced by the seed laser in the modulator. Thus, the transverse dispersion cannot be closed after the density modulation, which may cause FEL gain reduction in the radiator due to the increased beam size. In order to ensure adequate gain in the radiator, the dispersion induced beam size is required to be less than the intrinsic horizontal beam size contributed by the $\beta$ function. Then a dispersion $\eta$ up to 1 m is reasonable for SXFEL. According to Eq. (7), even with emittance of $\varepsilon_x = \varepsilon_y = 1.0 \mu m rad$ and $\eta = 1 m$, the beam energy spread can be effectively cooled from 84 to 15 keV, and the 30th harmonic bunching from the theory and 3D simulation is about 14%. Under such circumstances, Fig. 5(b) demonstrates that the initial bunching drives a coherent growth in the following 8.8 nm radiator, and the peak power saturates above 250 MW in six segments of a 3-m long undulator.

Now we consider the coherent harmonic generation setup on synchrotron radiation facilities, also known as optical klystron, in which the large energy spread is the main obstacle on the way to a short wavelength. We take the UVSOR-II storage ring parameters as an example [33], i.e., 0.6 GeV beam energy, 0.6 MeV energy spread, total geometry emittance of 17.5 nmrad, and coupling of 3%. Considering the rather small beam emittance in the vertical plane, each electron bunch was proposed to be vertically dispersed [28] only after it undergoes sufficient damping, and an 800 nm laser together with a 20 period modulator undulator are used to induce a maximum energy

FIG. 4 (color online). Bunching factor vs harmonic number at the exit of the dispersive chicane in the standard HGHG and the cooled HGHG configurations.

FIG. 5 (color online). The cooled-HGHG performance with respect to the normalized transverse emittance. (a) 30th harmonic bunching factor as a function of the horizontal emittance $\varepsilon_x$, while it is independent on the vertical emittance $\varepsilon_y$. (b) 8.8 nm radiation gain curve in the radiator, with the electron beam of $\varepsilon_x = \varepsilon_y = 1.0 \mu m rad$ and $\eta = 1 m$. 

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modulation of 1.2 MeV. Then with an optimal relation of $\alpha \eta \approx 6.5$ from the 3D simulation, the bunching factor of the sixth harmonic is enhanced to 23.0% in the cooled HGHG from 1.8% in the conventional setup, and thus the intensity of the 133 nm extreme ultraviolet radiation will be enhanced by 2 orders of magnitude in the following undulator. It should be pointed out that one could introduce a smaller energy modulation to get sufficient density modulation at a particular wavelength by using the cooled HGHG, which will effectively avoid the radiation pulse stretch by the microwave instability, and thus offer a better time resolution for users.

In summary, we proposed a new mechanism to remarkably cool the electron beam energy spread by off-resonance laser modulation [34]. It is demonstrated with theoretical analysis and numerical simulations that the proposed technique holds great prospects in frequency up conversion based on high brightness linacs and storage rings. There are several practical physical effects that were not included in these simple considerations. They include the method of generating the required dispersion, coherent and incoherent synchrotron radiation effects in dispersive elements, and the transverse gradient imperfection of the modulator undulator. These and other effects should be taken into account before carrying out a proof-of-principle experiment. It is worth stressing that the proposed method opens up the possibility of coherent hard x-ray generation by two-stage cooled HGHG. One may also cascade a cooled HGHG with a standard HGHG to preserve the FEL gain of the final short-wavelength radiation, as the beam size of the “fresh bunch” can be recovered by another dogleg after the first stage. Furthermore, this mechanism can be easily applied to laser-plasma accelerator based light sources.

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